Re-Thinking Risk
What the Beta Puzzle Tells Us about Investing

David Cowan, Sam Wilderman

Introduction

One cornerstone of finance theory is that investors demand return in exchange for assuming risk. As a consequence, the long-term returns of an investment strategy should be commensurate with the risks the strategy takes. This proposition sounds reasonable and intuitive, but it remains controversial. As both academics and practitioners have noted, there appear to be some anomalies, i.e., investment strategies that generate returns greater than expected in light of their perceived risks. One example that has generated considerable discussion recently in both academic and practitioner circles is what one might call the “beta puzzle”: portfolios of low beta stocks have historically matched or beaten broader equity market returns, and have done so with significantly lower volatility. At the same time, high beta stocks have significantly underperformed, exhibiting lower returns while appearing to take much more risk.1

A number of strategies have been proposed to take advantage of this perceived inefficiency, from risk parity to minimum-variance portfolios. In each case, the investment thesis hinges on the belief that the counterintuitive performance of high and low beta stocks is an exploitable anomaly.2 In this paper we argue that this puzzling phenomenon is not an anomaly at all, but, more simply, stems from a misunderstanding of risk. In examining the beta puzzle and what lies behind it, we provide a framework for thinking about risk and return that can offer insight into a wide variety of investment strategies, from low-beta equity portfolios to levered ETFs and hedge funds.

The Puzzle

Let’s begin by taking a look at the empirical data behind the beta puzzle. Exhibit 1 shows the performance of portfolios of high and low beta large-cap U.S. stocks.

As Exhibit 1 shows, the portfolio of low beta stocks has outperformed the broader market, with substantially lower realized volatility and smaller drawdown. The high beta portfolio has underperformed the market, and done so with substantially higher volatility and larger drawdown. This is the essence of the beta puzzle: why should the low-risk asset have no compensating reduction in performance, while the high-risk asset (which should be compensated for its additional risk) underperforms?

This phenomenon is not limited to the U.S.; the same pattern can be seen for global equities (Exhibit 2). As in the U.S., the global low beta portfolio provides a much better return than high beta despite appearing to carry much lower risk.

Proposed Explanation(s)

This puzzle has been widely discussed, and several explanations have been offered. To us, the most interesting of them theorizes that high beta assets trade at a premium because they provide implicit leverage to investors who are

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1 Here and throughout this paper we use beta in the usual way: as a measure of the sensitivity of an asset’s return to the contemporaneous return of the market. We measure this using 250-day returns of each stock, regressed against 250-day returns of the relevant universe, weighted by market capitalization.

2 In some cases the focus is more on volatility than beta, but in general the dynamics are similar. Beta is an important contributor to a stock’s total volatility, and becomes an even more central factor in contexts like minimum-variance portfolios, where the strategy is focused on low portfolio-level volatility.
Exhibit 1: Beta Puzzle – U.S.

<table>
<thead>
<tr>
<th></th>
<th>Universe</th>
<th>Low Beta</th>
<th>High Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>9.8%</td>
<td>10.6%</td>
<td>7.2%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>16.0%</td>
<td>12.5%</td>
<td>24.5%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-50.3%</td>
<td>-39.5%</td>
<td>-84.4%</td>
</tr>
</tbody>
</table>

Note: The universe is top 1,000 U.S. stocks; low and high beta portfolios are quartiles of 250-day beta within that universe (with betas regressed against that universe). All three portfolios are formed monthly and weighted by market capitalization.

Exhibit 2: Beta Puzzle – Global

<table>
<thead>
<tr>
<th></th>
<th>Universe</th>
<th>Low Beta</th>
<th>High Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>8.5%</td>
<td>10.1%</td>
<td>4.1%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>16.0%</td>
<td>11.9%</td>
<td>24.6%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-56.9%</td>
<td>-44.2%</td>
<td>-80.2%</td>
</tr>
</tbody>
</table>

Note: The universe is top 2,000 global equities by market capitalization; low and high beta portfolios are quartiles of 250-day beta within that universe, with each stock regressed against the performance of that universe within the stock’s region. All three portfolios are formed monthly and weighted by market capitalization.
not able to get it explicitly. High beta stocks, the theory goes, provide exposure similar to borrowing money to lever a market position—but the mandates of some investors prohibit them from borrowing money (or they may simply prefer to avoid it), and these investors might turn to high beta stocks to get their additional market exposure. This extra demand drives up the price of high beta stocks, driving down their long-term returns.

In our view, leverage is an important part of the explanation, but the offered theory misses a critical element. It is not the implicit nature of the leverage that is at the heart of the matter, but rather the fact that the leverage high beta provides comes with an additional benefit: it is leverage with protection.

Let's look at a stylized example to understand this feature a little more fully.

**Exhibit 3: Not All Leverage Is Created Equal**

To illustrate the desirability of protected leverage, Exhibit 3 contrasts how we might expect two forms of leverage to behave. Compare, for example, the fate of two investors, each starting with $100: one invests $100 in a portfolio of beta-2 stocks, while the other borrows an additional $100 from his prime broker and buys $200 of the market. The first is using implicit leverage, while the second leveres the portfolio explicitly. While the investors earn similar returns in a market advance, each gaining twice what the market does, their outcomes should differ when the market falls. When the market declines 50% or more, the investor who borrowed money explicitly has no capital left from his investment (and could potentially lose even more). The owner of the high beta portfolio, however, will have some value left in his portfolio, as the prices of the equities in the beta-2 portfolio will most likely remain above zero in a down 50% market (and they certainly cannot go down 120% in a down 60% market).

The point here is that the form of leverage offered by high beta is different in an important way from explicit borrowing. Investors should prefer this kind of leverage, and, in an efficiently priced market, they will accept a lower return for it. As we will show, the performance of high beta is not a product of excessive demand, but rather a reasonable and rational consequence of the fact that it provides a convex payoff to the market.

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Asymmetric Payoffs: Convexity/Concavity

We argue above that there are intuitive reasons to think that high beta stocks should provide different exposures in big up markets and big down markets. As we move from our stylized example to empirical results, we see that this is exactly what has happened over the past 40 years. The scatter plot below shows the 20-day performance of high beta stocks relative to the broad universe.

Exhibit 4: High Beta

![Scatter plot showing high beta return versus market return](image)

Source: GMO     As of 10/28/11

**Note:** High beta and universe are the same as those plotted in Exhibit 1; returns are rolling 20-day compound returns. The curve shown is a best-fit quadratic, and the convexity can be seen in the quadratic coefficient (0.6) in the fit equation, \( y = 0.6x^2 + 1.4x - 0.005 \). Since that term is positive, the payoff is convex. All terms in this fit are statistically significant (adjusting in the standard way for the overlapping periods) to the 1% level.

When we fit a curve to this data, we see that high beta stocks have a convex profile when compared against the market. This is exactly the characteristic we expect to see in protected leverage: the investor gets the extra upside participation, but in a down market he suffers less than he would using standard leverage.

Interestingly, low beta gives the opposite (i.e., concave) profile (Exhibit 5). Whereas high beta gives more upside than downside, low beta gives more downside than upside.

The asymmetry of these payoffs can also be seen in the realized betas in various market regimes. Exhibit 6 shows the realized betas of these portfolios in three different return environments. In significant up markets, the betas of these portfolios differ greatly, with the high beta portfolio providing nearly three times the market exposure of the low beta portfolio. In down markets, however, the betas of both portfolios are much closer to 1.

Borrowing from the language of the options market, high beta's convex payoff looks a lot like what an investor would get from owning the market plus a call on the market: in a rally you get the extra payoff of the call option, but in a decline you lose only your call premium. Low beta, on the other hand, is analogous to owning the market and selling a call option on the market, i.e., a covered call strategy. In big down markets you get to keep the option premium, but are otherwise fully exposed, while in up markets you get very little participation because you have sold off the upside in exchange for the call premium.
Exhibit 5: Low Beta

Note: See note to Exhibit 4; the quadratic coefficient in the low beta fit equation, $y = -0.6x^2 + 0.6x + 0.004$, is negative, indicating a concave payoff. The terms in this fit are also statistically significant to the 1% level.

Exhibit 6: Realized Betas

Notes: Realized betas are the slopes of best fit lines using the data points in Exhibits 4 and 5, restricted to points where the 20-day universe return falls into the indicated intervals.

Return = 20-Day Rolling Compound Return; Market = Top 1000 U.S. Equity Stocks by Market Capitalization; High Beta = Top 25% of Beta by Market Cap in Top 1000 U.S. Equities and Low Beta = Bottom 25% of Beta by Market Cap in Top 1000 U.S. Equities.
The convexity embedded in high beta stocks is a good thing. If it came for free, investors would always choose more exposure to up markets than down markets. The problem is that convexity, like most good things in life, comes at a price. To understand this better, we turn to instruments whose purpose is to give access to asymmetric payoffs: options.

Options

While the convexity of high beta is implicit, it is an explicit characteristic of options. A call option by its very nature exposes an investor to the upside, while losses are limited to the premium paid. So, you should logically expect a strategy that buys call options on the market to have greater participation in up markets than down markets. And of course, that is exactly what we see in Exhibit 7.

Exhibit 7: Buy 1 ATM Call with 1-Year Expiry

Above we show the trailing 20-day return from owning a one-year, at-the-money call option on the S&P 500 compared to owning the market itself. As expected, the exposure is greater on the upside than the downside.

As with high beta, this sort of asymmetric payoff is an attractive feature. But it does not come for free. To illustrate the cost of this asymmetry, Exhibit 8 displays the cumulative performance since 1996 (the period for which we have robust options data) of owning the market and owning a 1-year call on the market. While the market doubled over this period of time, the call buying program returned less than 50%, despite being "exposed to all of the upside in the market." This return differential may seem puzzling until you consider the risk differential in each approach: since the downside is capped at premium spent, owning the call is significantly safer than owning the market and, as a result, should earn a lower return.

One of the nice things about options is that they give you many ways to structure your market exposure. Above we looked at buying a call set to expire in a year. But we can examine options with different dates of expiry. By shortening the time to expiry, you reduce the time value of the option, but you also lower the amount of premium you have to spend. And, as you reduce the premium, you reduce the downside exposure of the strategy. It follows logically that this should reduce the return you expect to get paid. As Exhibit 9 shows, this is what we have observed over the past
Exhibit 8: Performance of Call-Buying

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>12-Mo. Calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>5.7%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>16.3%</td>
<td>9.4%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-50.9%</td>
<td>-34.3%</td>
</tr>
</tbody>
</table>

Source: OptionMetrics IvyDB, GMO As of 9/30/11

Note: Same portfolio as shown in Exhibit 7. No commissions, options priced at mid-point.

Exhibit 9: Performance of Call-Buying

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>1-Mo. Calls</th>
<th>3-Mo. Calls</th>
<th>6-Mo. Calls</th>
<th>12-Mo. Calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>5.7%</td>
<td>-1.2%</td>
<td>0.3%</td>
<td>1.7%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>16.3%</td>
<td>8.6%</td>
<td>8.5%</td>
<td>8.8%</td>
<td>9.4%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-50.9%</td>
<td>-46.7%</td>
<td>-41.4%</td>
<td>-36.7%</td>
<td>-34.3%</td>
</tr>
</tbody>
</table>

Source: OptionMetrics IvyDB, GMO As of 9/30/11

Note: Same as Exhibit 8, only expiry is varied. No commissions, options priced at mid-point.
15 years: the less you were exposed to market risk (i.e., the shorter the time to expiry and thus the lower the premium paid), the less return you would have generated.

Another way to structure equity exposure through options is by selling puts on the market. As opposed to buying calls, where you pay a premium for upside exposure, selling puts exposes you to all of the downside in exchange for collecting a premium. Exhibit 10 shows the performance of the put-writing versions of the call-buying strategies we show above.

Exhibit 10: Performance of Put-Selling

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>1-Mo. Puts</th>
<th>3-Mo. Puts</th>
<th>6-Mo. Puts</th>
<th>12-Mo. Puts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>5.7%</td>
<td>10.0%</td>
<td>8.2%</td>
<td>6.7%</td>
<td>5.2%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>16.3%</td>
<td>11.0%</td>
<td>10.0%</td>
<td>9.5%</td>
<td>9.3%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-50.9%</td>
<td>-29.8%</td>
<td>-28.1%</td>
<td>-28.2%</td>
<td>-31.5%</td>
</tr>
</tbody>
</table>

Note: See notes to Exhibits 7 and 8; portfolios are the same, except these have written puts rather than purchased calls. No commissions, options priced at mid-point.

Here again, the total returns are commensurate with the downside risks being taken: the short-put positions have much higher total returns than the long call positions. And, again, there is a relationship between the term of the options and the performance. Selling shorter-dated puts performs better than selling longer-dated ones because a short-dated put generally has less premium cushion than a long-dated put, and consequently suffers more in a sharp market decline.

The options market delivers a payment to investors willing to take downside exposure without upside (selling puts), but charges a payment from those who want upside without the downside (buying calls). And the more asymmetric the payoff, the bigger the payment gets. Short-dated puts and calls have very asymmetric payoffs—the most concave and convex, respectively—so the payment is highest in those positions. Consequently, these are the best- and worst-performing of the option strategies above.

It is important to note that, just like low beta, the put-writing strategies above have lower volatility and smaller drawdowns than the market, while still delivering good performance. But it is clear there is no anomaly here—although the simple realized risk statistics are low, put-writing carries the same downside risk as the market. Put-writing strategies have lower volatility because of the reduced exposure to market rallies, and they have smaller drawdowns because, in times of market stress, the premium paid to providers of insurance is very high. Concave strategies frequently have lower realized risk characteristics than convex ones, which can make them seem safer, but this does not mean that they are actually running less risk.
Back to the Puzzle

We have seen that high and low beta portfolios exhibit asymmetric, option-like payoffs, and we have also seen that in the options market, those different payoffs generate very different long-term returns. Let's now link these two ideas together to better understand the performance of high and low beta portfolios. To start, Exhibit 11 displays the performance of high and low beta since 1996 within the top 200 stocks in the U.S. by market capitalization (this is the period and universe for which we have the richest options coverage in our database).

Exhibit 11: Beta Puzzle since 1996

<table>
<thead>
<tr>
<th></th>
<th>Universe</th>
<th>Low Beta</th>
<th>High Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>5.9%</td>
<td>6.8%</td>
<td>3.8%</td>
</tr>
<tr>
<td>Annualized Volatility</td>
<td>16.4%</td>
<td>11.7%</td>
<td>28.5%</td>
</tr>
<tr>
<td>Maximum Drawdown</td>
<td>-49.8%</td>
<td>-33.2%</td>
<td>-83.2%</td>
</tr>
</tbody>
</table>

Source: OptionMetrics IvyDB, GMO As of 10/31/11

Note: The universe is top 200 stocks in the U.S. by market capitalization; the beta is regressed to the top 1,000, but both high and low beta portfolios are quartiles within the top 200 universe. All portfolios are formed monthly and weighted by market capitalization.

Even over this limited data set, the basic “puzzle” characteristics remain: low beta has delivered higher returns with substantially lower volatility. As we see in Exhibit 12, high beta's convex characteristic also persists.

In our look at the options market, we saw that convex payoffs are generally expensive, so it is reasonable to suspect that it is this convexity that leads to high beta's underperformance. To check this, we can examine what happens if we run a covered call program on the high beta portfolio, selling the desirable upside exposure back to the market in exchange for option premium. Doing this transforms high beta's convex payoff into a more linear one (Exhibit 13).

This payoff transformation should not be surprising. However, as can be seen in Exhibit 14, the cumulative performance change that results from this transformation is striking. Once we switch to covered calls, owning the high beta portfolio exhibits very similar risk and return characteristics to the market. And this makes perfect sense: as we have seen, the payoff to high beta looks like the market plus a call on the market—if we sell that call, we are left with the market.4

So, the underperformance of high beta can be understood as a premium payment for the call-like asymmetric payoff it offers. Once we give up that payoff and accept a more symmetric exposure, we earn returns very much like the market. The options market clearly indicates that the pricing of high beta is consistent with the asymmetric upside payoff that portfolio offers. Far from being an anomaly, in fact, high beta performs exactly the way it should.

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4 Note also that by adding the covered calls, we have lowered the realized volatility and drawdown of high beta, while increasing its return. The higher-than-market realized risk characteristics of high beta stem from the extra upside these stocks offer, not from their downside.
Having seen what a covered call program does for high beta, it is natural at this point to wonder what happens if we do the same with low beta (Exhibit 15).

Interestingly, there is no long-term performance improvement in the low beta case; whereas high beta was helped significantly by overwriting, low beta is not helped at all. The options market is willing to pay a hefty premium for
Exhibit 14: High Beta Covered Call

<table>
<thead>
<tr>
<th></th>
<th>Universe</th>
<th>High Beta</th>
<th>Covered Call</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annualized Return</strong></td>
<td>5.9%</td>
<td>3.8%</td>
<td>6.3%</td>
</tr>
<tr>
<td><strong>Annualized Volatility</strong></td>
<td>16.4%</td>
<td>28.5%</td>
<td>19.2%</td>
</tr>
<tr>
<td><strong>Maximum Drawdown</strong></td>
<td>-49.8%</td>
<td>-83.2%</td>
<td>-55.2%</td>
</tr>
</tbody>
</table>

Source: Ivy DB, GMO As of 10/31/11

**Note:** The universe is top 200 stocks in the U.S. by market capitalization; the beta is regressed to the top 1,000, but both high and low beta portfolios are quartiles within the top 200 universe. All portfolios are formed monthly and weighted by market capitalization. The covered call portfolio is the same as that in Exhibit 13. All options are priced at the far side of the spread, and performance includes the impact of commissions and fees on the options.

Exhibit 15: Low Beta Covered Call

<table>
<thead>
<tr>
<th></th>
<th>Universe</th>
<th>Low Beta</th>
<th>Covered Call</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annualized Return</strong></td>
<td>5.9%</td>
<td>6.8%</td>
<td>6.2%</td>
</tr>
<tr>
<td><strong>Annualized Volatility</strong></td>
<td>16.4%</td>
<td>11.7%</td>
<td>9.1%</td>
</tr>
<tr>
<td><strong>Maximum Drawdown</strong></td>
<td>-49.8%</td>
<td>-33.2%</td>
<td>-24.3%</td>
</tr>
</tbody>
</table>

Source: Ivy DB, GMO As of 10/31/11

**Note:** The universe is top 200 stocks in the U.S. by market capitalization; the beta is regressed to the top 1,000, but both high and low beta portfolios are quartiles within the top 200 universe. All portfolios are formed monthly and weighted by market capitalization. The covered call portfolio is formed the same way as in Exhibit 13, but on the low beta portfolio. All options are priced at the far side of the spread, and performance includes the impact of commissions and fees on the options.
the upside offered with high beta, but willing to pay nothing for the upside offered by low beta. And that is consistent with what we know about those portfolios: high beta offers a good bit of extra upside, worthy of a premium, but low beta offers little upside exposure, so it should not be able to command a premium for it.

Overall, then, the options market prices asymmetric payoffs in a way broadly consistent with what we see from high and low beta portfolios. Rather than interpreting the performance of these portfolios as an anomaly that violates the theory of efficient markets, it is simpler, and more consistent with the data, to explain these phenomena as a rational outcome of the asymmetric payoffs they offer.

**More Convexity: Levered ETFs**

We have seen how convex and concave payoffs play an important role in the performance of high and low beta stocks, as well as options. Another interesting place to see these effects is in the behavior of levered ETFs. Like high beta portfolios, these instruments exhibit performance that can seem puzzling, but which makes more sense when understood through the lens of convexity.

Levered ETFs are designed to provide a simple form of levered exposure to an underlying index. For example, the ProShares Ultra S&P 500 ETF (SSO) aims to give 2x the daily return of the S&P 500. On a day when the S&P 500 is up 5%, SSO aims to be up 10%, and when the S&P 500 is down 5%, SSO aims to be down 10%. As a consequence, SSO runs the risk of total loss; a market fall of 50% or more on a single day should bankrupt the instrument. Owners of the ETF cannot lose more than their invested capital, of course, but the value of their investment can go to zero in a big enough market decline.\(^5\)

Over time, though, the situation changes because every day the ETF manager has to rebalance the portfolio to be 200% exposed. For example, suppose the market goes down 30% one day and then another 30% a second day, for a total loss of 51%. The SSO returns will be -60% the first day and -60% the second day, so the two-day return will be -84%. Not pretty, but not a bankruptcy event. As long as the 50%+ market decline does not happen on a single day, the ETF investor will not lose all of his capital. So, on a multi-day basis, the levered ETF provides a form of protected leverage.

A look at the data reveals that SSO does indeed exhibit the characteristics we expect from protected leverage. Exhibit 16 shows the 20-day returns of SSO vs. S&P 500. We also show the corresponding data for SDS, a levered ETF that gives 2x the negative daily return of the S&P 500 (Exhibit 16).

While these two levered ETFs are opposites in terms of direction, with one giving long exposure and the other short, they both provide convex multi-day payoffs. In each case, the loss when the market moves the wrong way is not quite as bad as one might expect, while the gains when the market goes in the ETF’s favor are generally better.

Like high beta and other convex assets, however, the long-term performance of SSO and SDS reflects the cost of this convexity (Exhibit 17). Over a period when the S&P 500 is close to flat,\(^6\) an instrument aiming to give 2x leverage has lost almost 50%. As bad as that sounds, though, the situation on the short side is even worse: despite having the direction right, the instrument aiming to give 2x short leverage has also lost 50%. Whether you chose long or short, if you owned levered ETFs over this period, you lost. This performance can certainly seem puzzling, and some have interpreted it as evidence that these instruments are downright evil. But at this point we are in a position to see that this performance hit is a natural consequence of the convex payoffs levered ETFs provide.\(^7\)

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5 As a consequence, some levered ETFs do try to position their portfolios to prevent bankruptcy in a big single-day move. That protection is at the very extremes of price movements and generally does not affect the dynamics we describe here.

6 2008-present; this is the period over which we have listed option data on these two securities.

7 As with owning high beta stocks and/or buying call options, of course, bad performance is bad performance, whatever the source. Investors who do not want to pay the premium for a convex payoff should think twice before using these instruments. In addition, there are some other reasons to be concerned about levered ETFs (e.g., the fees they charge, the collateral they take in, and the impact their rebalancing trades have on the broader market). Our point here is simply that even if you leave those issues aside, the basic practice that a levered ETF engages in will produce a convex payoff, and in general investors should expect to pay a premium for that.
With high beta, we showed that once the payoff profile is transformed, puzzlingly bad returns become far more reasonable. Here too we can overwrite the levered ETFs and see what that does to the symmetry of their return profiles and to the resulting cumulative performance (Exhibits 18 and 19).
As Exhibits 18 and 19 demonstrate, by overwriting SSO and SDS and thereby forgoing upside exposure for a premium, we can easily transform the convex payoffs into concave ones. Of course, we are losing a very desirable characteristic, but in exchange we recoup a lot in longer-term performance.
Once again, giving up convexity leads to better performance since we get to keep the healthy premium we otherwise would have paid out.

**Concavity: Hedge Funds**

We have seen that the convexity/concavity of an investment strategy is important to understanding that strategy’s performance. Another area where we find this lens useful is in the context of hedge funds. Exhibit 20 shows how the monthly returns for a broad index of hedge funds compare to the S&P 500.

**Exhibit 20: Monthly Returns to HFRI Fund-Weighted Index**

The pattern is clear: hedge funds generally have a concave profile, losing more in down markets than they make in up markets. And from a performance point of view, this makes sense. As we have seen, convexity tends to be expensive, driving down performance, while concavity can add to returns. Since hedge funds are trying to make money, we should not be surprised to find them on the concave side of this trade. This result also matches conventional wisdom about risks becoming correlated in times of stress. To the consternation of many investors, strategies that offer diversification in normal market environments have turned out to be much more aligned in market declines.\(^8\)

As we have seen with high beta and levered ETFs, we can alter such asymmetric profiles through simple options strategies. In the earlier cases we removed convexity by over-writing call options. In order to remove the concavity from hedge funds, however, we have to buy downside protection – i.e., hedge them by buying puts. With one-month, at-the-money put options, we need to hedge about 60% of our hedge fund exposure to take out the downside exposure to the market; Exhibit 21 shows how this alters the payoff.

We are left with a strategy that has very little beta, and no convexity/concavity. Not surprisingly, the long-term returns suffer quite a bit once we make this adjustment (Exhibit 22).

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\(^8\) You may wonder how much of this effect is driven by the outlier months in the lower-left part of the scatter—September/October 2008 (U.S. financial crisis) and August 1998 (collapse of LTCM). These are well-known instances when both the equity market and hedge funds suffered substantial losses. Certainly they are significant data points, but in fact the concavity remains even when we exclude these months.
There are two important takeaways here. First, hedging your hedge fund portfolio is costly because that hedging program has to buy convex payoffs, and convex payoffs are expensive. The return of the HFRI index over this period is 9.0%, but the hedged version earns just 4.2% (just barely better than cash). In fact, the return stream of the hedging program used here, looked at as a stand-alone investment, is pretty painful (Exhibit 23).
Second, this result tells us something about what is really driving hedge fund returns: hedge funds get paid for taking downside market risk. Take away that risk, and you take away much of the return. In fact, a recent academic paper has made a strong case that hedge fund returns can be closely replicated by a simple put-writing strategy. In Exhibit 24 we show our version of this replication strategy, before and after a basic estimate of fees.

**Exhibit 24: Replicating Hedge Fund Performance**


9 Stafford and Jurek, *The Cost of Capital for Alternative Investments*, 2011. The replication is a 2x levered constant-delta, OTM put-writing strategy. Fees are assumed to be 3%/year, meant to capture the overall impact of fees on the order of 2 and 20.
These results suggest that hedge funds as a group earn steady returns by underwriting extreme downside market moves. Remarkably, an asset class that purports to be an “alternative” source of returns, with low correlation to equity markets, turns out to be simply another way to take downside equity market risk. Individual funds and strategies certainly vary greatly, but in aggregate, once fees are taken into account, hedge funds appear to offer nothing beyond a way to sell insurance against sharp market declines. This is a perfectly reasonable way to earn a return, but from a risk perspective offers less diversification than many investors expect.

In addition, the returns required to justify the real risks being taken by hedge funds are higher than people usually realize. When investors give hedge funds credit for having low beta, they will tend to think the funds are adding a lot of value, generating high returns relative to their risk. When they see them as earning a return for taking downside equity exposure, investors will tend to think the funds are generating a reasonable return, but nothing beyond what they should be getting given the risk they are taking.

Conclusion

The goal in this paper has been to explain the important role that asymmetric payoffs play in determining the risk and return characteristics of investment strategies. Investments with concave payoffs (buying low beta stocks, writing put options, and investing in hedge funds) generally have better long-term performance than those with convex payoffs (buying high beta stocks, buying call options, and owning levered ETFs). Altering the payoffs through the use of options also alters the long-term performance; adding convexity tends to hurt returns, while adding concavity tends to help.

There are three main implications. First, beware of beta as a measure of risk—when behaviors differ in up markets and down markets, simple beta will not be a good measure of the real risk in a strategy. This can cause one to see market inefficiencies where they are not really present, but more importantly, it can lead one to over-allocate to strategies that appear to be lower risk than they really are. Hedge funds, for example, carry a relatively low beta, but this understates substantially the real risks they are taking, and allocations to this type of investment should take that into account.

Second, a broad set of strategies exists for taking downside equity market risk, one of the very few risks that should earn a return over the long run. What separates these strategies from each other is not the risks they take, but the way they get paid for taking that risk. Like standard long equity positions, concave strategies like hedge funds, put-writing, and low beta equity portfolios all earn a return that can be traced to the downside market exposure they have. The volatility reduction and diversification these “alternatives” offer when placed alongside standard equity portfolios derive not from taking different risks, but rather from different payoffs when the market goes up.

Third, strategies that attempt to take away downside risk will generally prove very costly over any reasonably long period of time. Tail risk protection strategies, which have become quite popular in recent years, seek to provide insurance against big down markets by engaging in a range of activities that approximate (and sometimes explicitly include) buying puts on the market. Purchasing such protection on your portfolio might reduce your exposure to extreme down markets, and will likely help you sleep better at night, but will almost certainly come at the expense of long-term returns.

Asymmetric payoffs have important consequences for the performance of an investment strategy. Understanding the premium you will pay for convexity, and the return you can earn from concavity, is critical to structuring your investment program and generating good long-term returns.